Name: _____ AP Calc BC Practice Test Ch. 9A

1. Given the series $5+4+\frac{16}{5}+\frac{64}{25}+...$

   a. Write the series as a sum

   b. Find the sum if convergent

2. Write the MacLaurian Series for each of the following. Include the general term.

   a. $f(x)=\sin x$

   b. $f(x)=\tan^{-1} x$

3. Let $f(x)=\sin(x)$

   a. Use the MacLaurin polynomial of order 5 (3 nonzero terms) for $f(x)$ to approximate sin(0.2)

   b. Find the Lagrange Error Bound of the polynomial if $0\le x\le 0.2$

4. Write each function as a power series in summation form:

   a. $f(x)=\frac{x}{1+x^2}$

   b. $f(x)=\frac{5}{x}$

5. Let $g(x)=\frac{\cos\left(x^3\right)}{2x}$. Find each of the following.

   a. Write the 1st four nonzero terms and the general term for the Maclaurin series for cos x.

   b. Write the 1st four nonzero terms and the general term for the Maclaurin series for $g(x)$.

6. Let $f(x)$ be a function that has derivatives of all orders for all real numbers.

   If $f(0)=9$, $f'(0)=5$, $f''(0)=-4$, and $f'''(0)=36$, write the third order Taylor polynomial for $f(x)$ at x = 0 and use it to approximate $f(0.3)$.

7. Find the function for each power series and give the interval of convergence

   a. $\sum_{n=0}^{\infty} 2(x-1)^n$

   b. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{x}{4}\right)^{n-1}$

8. Given that $x-\frac{x^2}{2}+\frac{x^3}{3}+\ldots+\frac{(-1)^{n-1}x^n}{n}$ is a power series representation for $\ln(1+x)$.

Find a power series representation for $x^2\ln(1+x^3)$, including a general term for n.

9. Let $f(x)$ be a function that is continuous and differentiable at all real numbers and let $f(2)=3$, $f'(2)=5$, and $f''(2)=4$

a. Write a 2nd order Taylor polynomial for $f(x)$ centered at a = 2.

b. Given $g'(x)=f(x)$ and $g(0)=1$, write a 3rd order Taylor polynomial for $g(x)$ centered at a = 2.

c. If $f'''(x)\le 7$ for all x, find the Lagrange error bound for the approximation of $f(2.2)$

10. Find the Taylor polynomial of order 3 generated by $f(x)=\cos(x)$ at $x=\frac{\pi}{3}$.

11. Given the Maclaurin series for $f(x)$ is $2x+3x^2+\frac{4x^3}{2}+\frac{5x^4}{6}+\ldots+\frac{(n+1)x^n}{(n-1)!}+\ldots$

a) Find $f''(0)$

b) Let $g(x)=xf'(x)$. Write the Maclaurin series for $g(x)$. Include a general term for n.

c) Let $h(x)=\int_0^x f(t)dt$. Write the Maclaurin series for $h(x)$. Include a general term for n.